In this short note we explain how to deduce $|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$

This would follow from $|\vec{u}\times\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 - |\vec{u}|^2|\vec{v}|^2\cos^2\theta$

$$= |\vec{u}|^2|\vec{v}|^2 - |\vec{u}\cdot\vec{v}|^2$$

Note, if $\vec{u} = \langle a_1, b_1, c_1\rangle$ then $\vec{u}\times\vec{v} = \langle b_1c_2 - c_1b_2, c_1a_2 - a_1c_2,$
$\vec{v} = \langle a_2, b_2, c_2\rangle$ $a_1b_2 - a_2b_1\rangle$

so $|\vec{u}\times\vec{v}|^2 = (b_1c_2 - c_1b_2)^2 + (c_1a_2 - a_1c_2)^2 + (a_1b_2 - a_2b_1)^2$

$$= b_1^2c_2^2 + c_1^2b_2^2 + c_1^2a_2^2 + a_1^2c_2^2 + a_1^2b_2^2 + a_2^2b_1^2 - 2b_1b_2c_1c_2 - 2c_1c_2a_1a_2$$
$$-2a_1a_2b_1b_2.$$

On the other hand, $(\vec{u}\cdot\vec{v})^2 = (a_1a_2 + b_1b_2 + c_1c_2)^2$

$$= a_1^2a_2^2 + b_1^2b_2^2 + c_1^2c_2^2 + 2a_1a_2b_1b_2 + 2a_1a_2c_1c_2$$
$$+ 2b_1b_2c_1c_2.$$

So, $|\vec{u}\times\vec{v}|^2 + (\vec{u}\cdot\vec{v})^2$

$$= a_1^2a_2^2 + b_1^2b_2^2 + c_1^2c_2^2 + b_1^2c_2^2 + c_1^2b_2^2 + c_1^2a_2^2 + a_1^2c_2^2 + a_1^2b_2^2 + a_2^2b_1^2$$

$$= a_1^2(a_2^2 + b_2^2 + c_2^2) + b_1^2(a_2^2 + b_2^2 + c_2^2) + c_1^2(a_2^2 + b_2^2 + c_2^2)$$

$$= (a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2) = |\vec{u}|^2|\vec{v}|^2,$$ as desired.